

Department of Electrical Engineering

Instrumentation Lab

EE-702

Experiment No.-8 Design active low pass filters for instrumentation purpose with different types and order.

Objective: Design realization and testing of low pass filters with R, C components.

Equipment/Apparatus required CRO, Multi meter, 741 IC, Resistances, capacitor, power supply (0-15 V DC), breadboard, connecting wires, low frequency sine wave function generator.

Theory:

Apart from the distinction between low-pass, high-pass, band pass and band stop, filters are further divided according to the attenuation characteristic. This is due to the fact, that it is not possible to implement filters with an abrupt change from pass band to stop band behavior as it would be expected from an ideal filter. The transition between pass band and stop band is always continuous. Close approximations of the ideal filter can be achieved with Butterworth, Bessel, and chebychev types.

The Bessel filter does not show a very distinct transition between pass band and stop band but its advantage is in its phase response: the phase shift between input and output voltage is a linear function of the frequency, yielding a constant group delay in the pass band.

The Butterworth filter shows a frequency response which runs very flat over a large bandwidth and changes very shortly before the limiting frequency. Therefore it is said to have a maximumally flat filter characteristic.

With the same number of network elements, we can also implement filters with a more distinct transition between pass band and stop band. These are the chebychev filters. However, this advantage has to be paid by a ripple in the pass band.

Low-pass Butterworth Filters (Sallen key Approach):

Butterworth filters have the properties that

- (1) The response is "maximumally flat," which means that there is no ripple in the pass band.
- (2) As the order of the filter n gets larger, the filter characteristics approach those of the ideal low-pass filter.

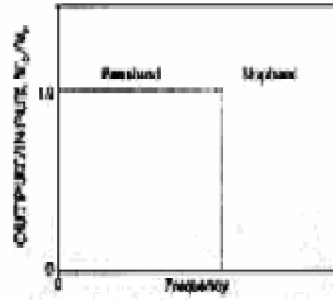


Figure 1: Ideal char. of low pass filter

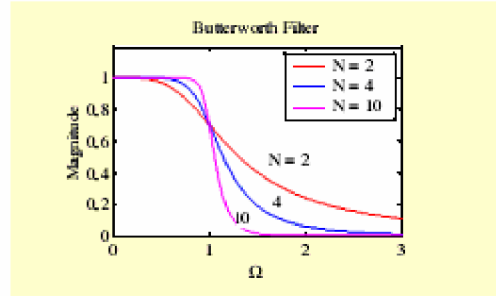


Figure 2: Char. of Butterworth low pass filter

The design begins by choosing four fiducial points. These points are:

- 1) Gain K_1 at frequency Ω_1 -low cut off point
- 2) Gain K_2 at frequency Ω_2 -upper cut off point

Assuming that the static (low-frequency) gain is unity, we thus want two conditions to hold. These are:

$$0 \geq 20 \log |H(j\Omega)| \geq K_1 \text{ for all } \Omega \leq \Omega_1$$

$$20 \log H(j\Omega) \leq K_2 \text{ for all } \Omega > \Omega_2$$

Where $H(j\Omega)$ is the transfer function which is given by

$$|H(j\Omega)|^2 = H(j\Omega)H(-j\Omega) = \frac{(A_{vo})^2}{1 + \left(\frac{\Omega}{\Omega_0}\right)^{2n}}$$

where A_{vo} is the mid band gain and Ω_0 is the cut off frequency.

Substituting these two conditions yields

$$10 \log_{10} \left[\frac{1}{1 + \left(\frac{\Omega_1}{\Omega_0}\right)^{2n}} \right] = K_1 \text{ (in dB)} \quad (1)$$

and

$$10 \log_{10} \left[\frac{1}{1 + \left(\frac{\Omega_2}{\Omega_0}\right)^{2n}} \right] = K_2 \text{ (in dB)} \quad (2)$$

from these

$$\left(\frac{\Omega_1}{\Omega_0}\right)^{2n} = 10^{-\left(\frac{K_1}{10}\right)} - 1 \quad (3)$$

and

$$\left(\frac{\Omega_2}{\Omega_0}\right)^{2n} = 10^{-\left(\frac{K_2}{10}\right)} - 1 \quad (4)$$

Dividing Equation (3) by Equation (4) yields

$$\left(\frac{\Omega_1}{\Omega_2}\right)^{2n} = \frac{10^{-\left(\frac{K_1}{10}\right)} - 1}{10^{-\left(\frac{K_2}{10}\right)} - 1} \quad (5)$$

Solving for n yields

$$n = \frac{\log_{10} \left[\frac{\left(10^{-0.1K_1} - 1\right)}{\left(10^{-0.1K_2} - 1\right)} \right]}{2 \log_{10} \left(\frac{\Omega_1}{\Omega_2} \right)} \quad (6)$$

The resulting value of n should be rounded up to the nearest whole number. This is the order of the filter to be designed and built. Using this value of n in Equations (3) and (4) will yield two different values of Ω_0 . The solution is to choose which set of fiducial points will be met exactly. For example, if we wish to satisfy the requirements at the first set of points (K_1, Ω_1), then we would solve for Ω_0 from

$$\Omega_0 = \frac{\Omega_1}{\left(10^{-\frac{K_1}{10}} - 1\right)^{\frac{1}{2n}}} \quad (7)$$

If on the other hand we wish to satisfy the requirements at the second set of points (K_2, Ω_2) then we would solve for Ω_0 from

$$\Omega_0 = \frac{\Omega_2}{\left(10^{-\frac{K_2}{10}} - 1\right)^{\frac{1}{2n}}} \quad (8)$$

From Table first-order transfer function is of the form

$$H(j\Omega) = \frac{A_{vo}}{j\Omega/\Omega_0 + 1} \quad (1.3)$$

Where Ω_0 is the half-power point (also known as the 3dB point, cutoff frequency, or break frequency) and A_{vo} is the low-frequency (static) gain. The gain of the first-order Butterworth filter rolls off with a slope of -20 dB/decade. The second-order Butterworth filter is of the form

$$H(j\Omega) = \frac{A_{vo}}{\left(j\Omega/\Omega_0\right)^2 + 2\xi\left(j\Omega/\Omega_0\right) + 1} \quad (1.4)$$

The filters can be realized with the following circuit

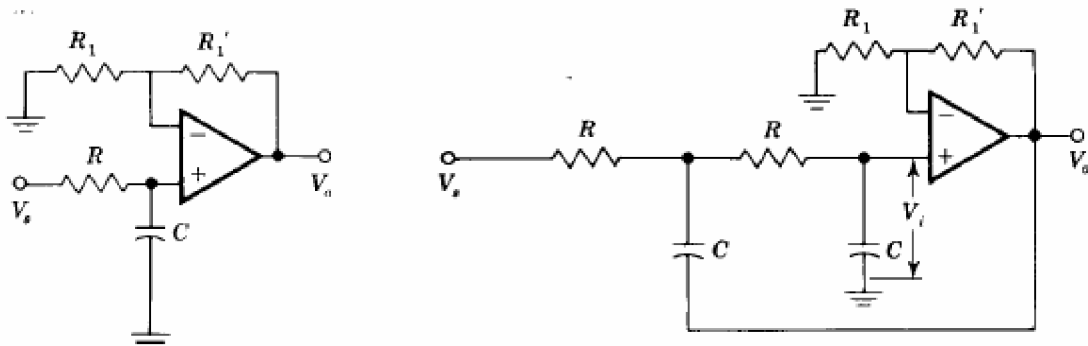


Figure 3- first-order and second order filter realizations

The cutoff frequency Ω_0 and gain A_{vo} are given by

$$A_{vo} = 1 + \frac{R_1'}{R_1} \quad \Omega_0 = \frac{1}{RC} \quad (9)$$

For a second-order filter, and if the cutoff frequency for a first-order filter is

$$\Omega_0 = \frac{1}{RC} \quad (10)$$

The value of A_{vo} for the first-order system is not important to the design, and can be any value. The damping ratio for the second-order frequency is found by solving Equation (11)

$$A_{vo} = 3 - 2\xi \quad (11)$$

Design Procedure:

The procedure for designing analog Butterworth filters is:

1. Identify the characteristics feature from given design specifications (K_1 , K_2 , Ω_1 , and Ω_2).
2. Calculate the required order of the filter n from Equation (6).
3. Calculate the required 3-dB point (cutoff frequency Ω_0) from either Equation (7) or Equation (8).
4. If the design includes a second-order filter, calculate the required mid-band gain A_{v0} from Equation (11). The damping ratio ξ is found from Table 1 by comparing second order polynomial. The feedback resistors are then calculated from Equation (9). If the design includes a first-order filter, the value of the mid-band gain is not important.
5. Pick a suitable value of C and calculate value of R from Equation (10).
6. Assemble the circuit (refer to figure 3) on the bread-board. Use a 5-volt DC power supply
7. Observe the output waveform on oscilloscope and record the reading in observation table according to specifications
8. If a hi-pass filter is desired, then switch the R and C components. If a band-pass filter is desired, then cascade a low-pass with a hi-pass filter. If a band-reject is desired, then add high-pass and low-pass filters.

Observation table:

Input voltage=------(V)

S.No.	Frequency(Hz)	Output (V)	Gain
1			
2			
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20			

Table 1 –Normalized Butterworth polynomials

n	Factors of polynomial $B_n(s)$
1	$(s + 1)$
2	$(s^2 + 1.414s + 1)$
3	$(s + 1)(s^2 + s + 1)$
4	$(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)$
5	$(s + 1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)$
6	$(s^2 + 0.518s + 1)(s^2 + 1.414s + 1)(s^2 + 1.932s + 1)$
7	$(s + 1)(s^2 + 0.445s + 1)(s^2 + 1.247s + 1)(s^2 + 1.802s + 1)$
8	$(s^2 + 0.390s + 1)(s^2 + 1.111s + 1)(s^2 + 1.663s + 1)(s^2 + 1.932s + 1)$

Chebyshev Filters:

Theory:

The Chebyshev filter class is based on Chebyshev Cosine polynomials

$$C_N(w) = \begin{cases} \cos(N \cdot \cos^{-1} w) & |w| \leq 1 \\ \cosh(N \cdot \cosh^{-1} w) & |w| > 1 \end{cases} \quad (12)$$

and we can use the following recursion to generate higher-order terms:

$$C_N(w) = 2w \cdot C_{N-1}(w) - C_{N-2}(w); \quad C_0(w) = 1; \quad C_1(w) = w \quad (13)$$

The resulting filter has a low-pass characteristic or order N:

$$|H(s)|^2 = \frac{1}{1 + \epsilon^2 C_N^2(w)} \quad (14)$$

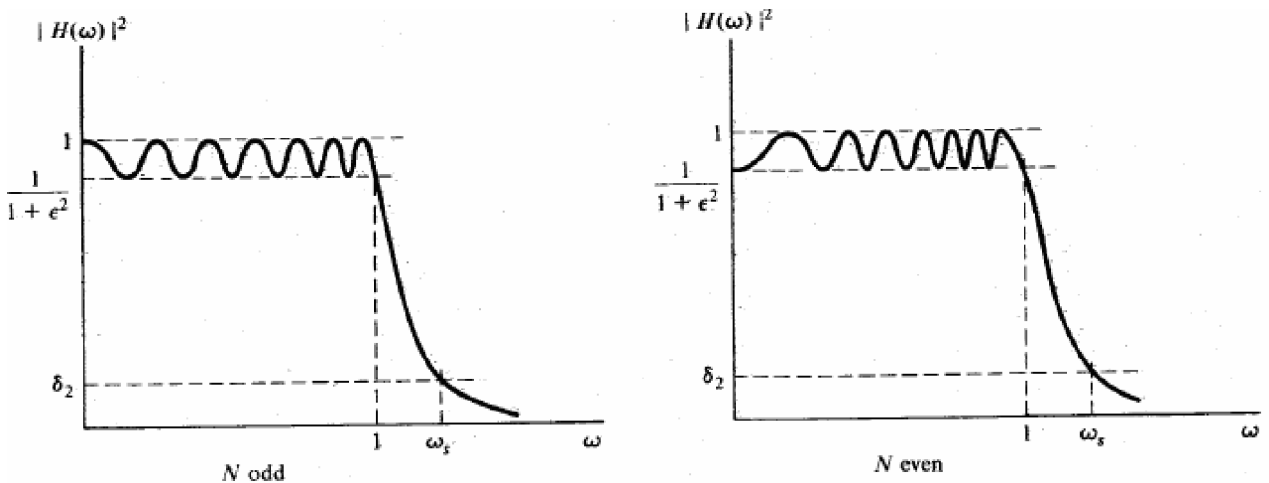


Figure 4: Magnitude characteristics:

$|H(s)|$ have ripples between 1 and $1/\sqrt{1+\epsilon^2}$ and for large values of w , stop-band region:

$$|H(w)| \cong 1/\epsilon \cdot C_N(w); \quad (15)$$

Attenuation- for lower cut off point

$$20 \cdot \text{Log}_{10} \epsilon + 20 \cdot \text{Log}_{10} C_N(w) \quad (16)$$

for upper cut off point

$$20 \cdot \text{Log}_{10} \epsilon + 6(N-1) + 20 \cdot N \cdot \text{Log}_{10} w \quad (17)$$

Let us introduce a new parameter β :

$$\beta = \frac{1}{N} \cdot \text{Sinh}^{-1}\left(\frac{1}{\epsilon}\right) \quad (18)$$

The poles of H(s) are :

$$s_k = \sigma_k + jw_k \quad \text{for } k = 0, 1, 2, \dots, N-1 \quad (19)$$

$$\sigma_k = \text{Sin}\left(\frac{2k-1}{N}\right) \cdot \frac{\pi}{2} \cdot \text{Sinh}(\beta) \quad \text{and} \quad w_k = \text{Cos}\left(\frac{2k-1}{N}\right) \cdot \frac{\pi}{2} \cdot \text{Cosh}(\beta) \quad (20)$$

which are located on an ellipse in the s-plane with an equation:

$$\frac{\sigma_k^2}{\text{Sinh}^2(\beta)} + \frac{w_k^2}{\text{Cosh}^2(\beta)} = 1 \quad (21)$$

The transfer function can be obtained by using poles calculated earlier. Higher order transfer function can be implemented by cascading 2nd-order blocks and 1st-order block. The op-amp circuits below are low-pass, unity-gain blocks with their corresponding transfer functions

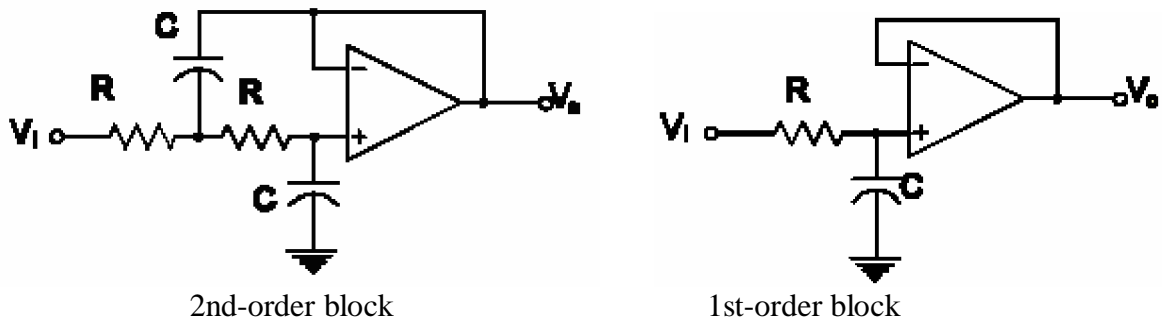


Figure5: configuration of 2nd and 1st order filter.

$$G(s) = \frac{1}{s^2 R^2 C_1 C_2 + 2sRC_2 + 1}$$

$$G(s) = \frac{1}{sRC + 1}$$

The R and C values of each block transfer function must satisfy the coefficients of the corresponding polynomial properly scaled in frequency, that is, with s divided by the cut-off frequency. Each block is designed independent of the others. In general, there are two types of polynomials in the denominator: first order and second order. Obviously, even-ordered filters do not include a first-order polynomial and do not require a first-order block. Design of a first-order block is straightforward since it only contains one s-term. First, divide the normalized 1st-order polynomial of the form (s + alpha) by alpha to obtain the new polynomial (βs + 1), where β=1/a; then replace s by s/w₀ and equate the term to sRC to solve for C as follows.

$$\beta \frac{s}{\omega_0} = sRC, \quad C = \frac{\beta}{\omega_0 R}$$

For the 2nd order blocks, equate coefficients for both s and s^2 terms. The following general steps describe the process for deriving the design formulas. Divide the normalized 2nd-order polynomial of the form $s^2 + b_1s + b_0$ by b_0 to obtain the new polynomial $a_2s^2 + a_1s + 1$. This puts it in the same form as that from the block transfer function given above. Replace s by s/ω_0 and equate resulting terms to those from the 2nd-order block transfer function.

$$a_2 \frac{s^2}{\omega_0^2} = s^2 R^2 C_1 C_2, \quad a_1 \frac{s}{\omega_0} = 2 s R C_2$$

Solve for C2

$$C_2 = \frac{a_1}{2 \omega_0 R}$$

Solve for C1

$$C_1 = \frac{a_2}{\omega_0^2 R^2 C_2}$$

Design Procedure:

1. From the pass band specification determine equation ϵ from (15)
2. Using the stop-band specification and (17) determine (N).
3. Calculate the poles and determine the transfer function.

First-order block design:

- Determine β from equation (18).
- Pick a suitable value of C.
- Calculate R to obtain the desired cut-off frequency in rad/sec

Second-order block design:

- § Obtain the coefficients a_2 and a_1 .
 - § Pick a suitable value for C2.
 - § Determine value of R for the desired cut-off frequency (rad /sec).
 - § Calculate C1 using the value of C2 and value of R just found.
4. Assemble the circuit diagram on bread-board (refer figure 5) by using values of R1,R2 and C as calculated.
 5. Observe the output waveform on oscilloscope and record the reading in observation table according to specifications.

Observation table:

Input voltage=------(V)

S.No.	Frequency(Hz)	Output (V)	Gain
1			
2			
-----	-----	-----	-----
20			

Simulation:

Simulate on PSPICE / ORCAD and extract main features and report. the points K_1 , Ω_1 , K_2 and Ω_2 and roll off rate. Assemble the design filter on bread –board and test for response

Results: Assemble the designed filter on bread-board and test for response.

Plot the frequency response of the filters on semi log paper. Compare the simulated and experimental results obtained with reference to design specifications.

Discussion:

1. What is frequency scaling? What are its benefits?
2. Why practical and theoretical results are having variations (if any)?How they can be accounted for?

Example1. Design a maximally flat Butterworth filter so that attenuation $K_1=0.0039$, $\Omega_1=1\text{KHz}$, attenuation $K_2=10$, $\Omega_2=10\text{KHz}$, are obtained.

Solution: Calculating n with the help of given formula

$$(1) \quad n = \frac{\log_{10} \left[\frac{(10^{-0.1K_1} - 1)}{(10^{-0.1K_2} - 1)} \right]}{2 \log_{10} \left(\frac{\Omega_1}{\Omega_2} \right)} = 2$$

$$(2) \quad \Omega_0 = \frac{\Omega_1}{\left(10^{\frac{K_1}{10}} - 1 \right)^{\frac{1}{2n}}} = 5776.05 \text{ Hz}$$

$$(3) \quad A_{v0} = 3 - 2\xi = 1.586$$

and

$$A_{v0} = 1 + \frac{R_1'}{R_1}$$

Choose $R_1 = 1\text{K} \Omega$, we get $R_1' = 583 \Omega$

$$\Omega_0 = \frac{1}{RC}$$

Choosing the value of $C = 0.1 \mu\text{f}$,
 $R = 1731.28 \Omega$

Assemble the circuit on bread board by using Figure 3.

Example2: Design a chebychev filter with attenuation not more than 1.0 dB for $|\omega| \leq 6280\text{rad/s}$ (1KHz) and at least 10 dB for $|\omega| \geq 31400 \text{ rads/s}$ (5KHz). Assume cut off frequency 18840 rad/s (3KHz).

Let us normalize the parameters by to get, $\omega_p = 1.0$ and $\omega_s = 5.0$

$$20 \cdot \text{Log}_{10} \frac{1}{\sqrt{1 + \varepsilon^2}} = -1 \Rightarrow \varepsilon = 0.509$$

$$10 = 20 \cdot \text{Log}_{10} 0.509 + 6(N - 1) + 20 N \cdot \text{Log}_{10} 5 \Rightarrow N = [1.092] = 2$$

$$\frac{s}{w_p} = -1/\sqrt{2} \pm j1/\sqrt{2}$$

$$w_p = 6280, \quad s_{1,2} = -3432.54 \pm j5607.53$$

$$H(s) = \frac{1}{(s + 3432.54)^2 + 5607.53^2}$$

By calculating the values of a_1 and a_2 :

$$a_1 = 0.000000023, \quad a_2 = 0.000158815$$

For cut-off frequency of 18840rad/s (3KHz)

Assume $C_2 = 0.001\text{pf}$

Then $R_1=R_2=R=3.83\text{K } \Omega$; $C_1 = 1202 \mu\text{f}$

With the design components, test the filter by simulation on PSPICE or ORCAD and note the features

Assemble the appropriate circuit on bread-board from Figure4, and test the response .